1976 AB2

$$
\begin{array}{r}
x^2 - 4 \\
x-3 \overline{\big) x^3 - 3x^2 - 4x + 12} \\
x^2 - 3x^2 \\
\hline
0 - 4x + 12 \\
-4x + 12 \\
\hline
0
\end{array}
$$

$f(x) = (x-3)(x^2-4)$
$= (x-3)(x-2)(x+2)$

a) The zeros of $f(x)$ are
$x = +2, -2, +3$.

b) $\lim\limits_{x \to 3} h(x) = \lim\limits_{x \to 3} \dfrac{f(x)}{x-3}$

$= \lim\limits_{x \to 3} \dfrac{(x-3)(x-2)(x+2)}{(x-3)}$

$= \lim\limits_{x \to 3} (x-2)(x+2) = (1)(5)$

$= 5$

The f^n will be continuous only if $p = 5$.

A fn is continuous at a pt $(x=3)$ if:
a) $h(3)$ is def^n (it is, it $= p = 5$)
b) $\lim\limits_{x \to 3} h(x)$ exists (it does, it $= 5$)
c) $\lim\limits_{x \to 3} h(x) = h(3)$ (it does).

c) $h(x) = \begin{cases} \frac{f(x)}{x-3} & , x \neq 3 \\ p & , x = 3 \end{cases}$

A f^n is Even if:

$f(-x) = f(x)$.

$h(3) = 5$

$h(-3) = \frac{(-3)^3 - 3(-3)^2 - 4(-3) + 12}{-6}$

$= -27 - 27 + 12 + 12$

$= \frac{-30}{-6} = +5$

so... consider $h(-a)$.

$h(-a) = (-a^3 - 3a^2 + 4a + 12)/(-a-3)$

$h(a) = (a^3 - 3a^2 - 4a + 12)/(+a-3)$

If $h(x)$ is even, these are equal...

$$\frac{-a^3 - 3a^2 + 4a + 12}{-(a+3)} = \frac{a^3 - 3a^2 - 4a + 12}{(a-3)}$$

$(-a^3 - 3a^2 + 4a + 12)(a-3) \stackrel{?}{=} (a^3 - 3a^2 - 4a + 12)(-(a+3))$

$-a^4 - 3a^3 + 4a^2 + 12a + 3a^3 + 9a^2 - 12a - 36 \stackrel{?}{=}$

$-(a^4 - 3a^3 - 4a^2 + 12a + 3a^3 - 9a^2 - 12a + 36)$

$-3a^3 + \cancel{12a} + 3a^3 - \cancel{12a} \stackrel{?}{=} 3a^3 - \cancel{12a} - 3a^3 + \cancel{12a}$

$0 = 0$ ∴ true for every x

$h(-1) = \frac{(-1)^3 - 3(-1)^2 - 4(-1) + 12}{-4}$

$= \frac{-1 - 3 + 4 + 12}{-4} = \frac{12}{-4} = -3$

$h(1) = \frac{1 - 3 - 4 + 12}{-2} = \frac{6}{-2} = -3$

∴ f^n is even

3.3/51) Show that

$$y = x^3 + 3x + 1 \quad \text{satisfies}$$

$$y''' + xy'' - 2y' = 0$$

Alg 2

$x^2 = 4$

called a "differential equation"

$$\boxed{\begin{aligned} y &= x^3 + 3x + 1 \\ y' &= 3x^2 + 3 \\ y'' &= 6x \\ y''' &= 6 \\ y^{(4)} &= 0 \end{aligned}}$$

$$y''' + xy'' - 2y' = 0$$

$$(6) + x(6x) - 2(3x^2 + 3) \stackrel{?}{=} 0$$

$$6 + 6x^2 - (6x^2 + 6) \stackrel{?}{=} 0$$

$$6 + 6x^2 - 6x^2 - 6 \stackrel{?}{=} 0 \; \checkmark$$

3-7/example 1) A tanker has a hole. Oil spills in a circular pattern. The radius of the spill increases at a rate of 2 ft/sec.
How fast is the area growing when the radius of the spill is 60 ft.

instantaneous rates of change... i.e. derivative

1) picture

2) create an equation that's true at every time (involving knowns and wants)

$$A = \pi r^2$$

I am thinking of A, and r, as functions of time. π is a constant

3) Differentiate with respect to t...

$$A = \pi r^2$$

$$\frac{d}{dt}(r^2) = 2(r)\frac{dr}{dt}$$

$$\frac{dA}{dt} = \pi\left(2r\frac{dr}{dt}\right)$$

4) only now, may I substitute in what I know...

$r = 60$ ft; $\frac{dr}{dt} = 2\frac{ft}{sec}$

$$\frac{dA}{dt} = \pi(2(60)(2)) = 240\pi \frac{ft^2}{sec}$$

Why use chain rule?

b/c $\frac{d}{dt}(r^2) = 2r\frac{dr}{dt}$ and NOT

$2r$

$[r = f(t)]^2$

$$\frac{d}{dr}(\pi r^2) = 2\pi r$$

$$\frac{d}{dt}(\pi r^2) = 2\pi r\frac{dr}{dt}$$

$f(x) = x^3 - 3x^2 - 4x + 12 = 0$

$$
\begin{array}{r|l}
 & x^2 \quad - 4 \\
\hline
x-3 & x^3 - 3x^2 - 4x + 12 \\
 & x^3 - 3x^2 \\
\hline
 & 0 \quad - 4x + 12 \\
 & \quad\;\; - 4x + 12 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$f(x) = (x-3)(x^2-4)$

$= (x-3)(x-2)(x+2)$

$x = 3, 2, -2$ A $f(3) = 0$

2

b) A f^n is cont at a pt (eg $x=3$) if:

a) $f(3)$ must be defined [and it is... $f(3) = p$]

b) $\lim_{x \to 3} h(x)$ exists.... and it does.

$$\lim_{x\to 3} h(x) = \lim_{x\to 3} \frac{x^3-3x^2-4x+12}{x-3} = \lim_{x\to 3} \frac{(x-3)(x-2)(x+2)}{(x-3)}$$

$$= \lim_{x\to 3} (x-2)(x+2) = (1)(5) = 5$$

c) $\lim_{x\to 3} h(x) = 5 = h(3) = p.$

b) $\therefore$ $h(x)$ is continuous at $x=3$ if $p = 5$

c) A fn $f(x)$ is even if

$$f(x) = f(-x)$$

for every x.

$h(3) = 5$

$$h(-3) = \frac{(-3)^3 - 3(-3)^2 - 4(-3) + 12}{(-3) - 3}$$

$$= \frac{-27 - 27 + 12 + 12}{-6}$$

$$= \frac{-30}{-6} = 5$$

$$h(a) = \frac{a^3 - 3a^2 - 4a + 12}{a - 3}$$

$$h(-a) = \frac{-a^3 - 3a^2 + 4a + 12}{-a - 3}$$

$$\frac{a^3 - 3a^2 - 4a + 12}{a - 3} \stackrel{?}{=} \frac{-a^3 - 3a^2 + 4a + 12}{-(a+3)}$$

$$(a^3 - 3a^2 - 4a + 12)(-(a+3)) \stackrel{?}{=} (-a^3 - 3a^2 + 4a + 12)(a - 3)$$

$$h(x) = \begin{cases} (x-2)(x+2) = x^2 - 4 \ ; & x \neq 3 \\ 5 \ ; & x = 3 \end{cases}$$

$h(a) = a^2 - 4$

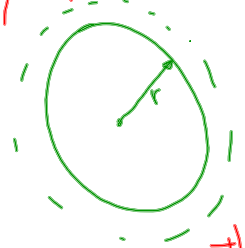
$h(-a) = (-a)^2 - 4 = a^2 - 4$

same

$\therefore h(x)$ is even

3.7/example 1 A tanker hits something and gets a hole. Oil leaks in a circular pattern with the radius of the circle increasing at a rate of 2 ft/sec. How fast is the area changing when the radius is 60 ft.

1) picture



2) create an equation that's true at every time. Ideally this involves only "knowns" and what you need to find out.

$$A = \pi r^2$$

Now we have to think of A and r as functions of time, t. π is a constant

3) Differentiate "FIND y'" of our eqn...

$$A = \pi r^2$$

$$\frac{d}{dt}(A) = \frac{d}{dt}(\pi r^2) = \pi\left(\frac{d}{dt}(r^2)\right)$$

$$\frac{dA}{dt} = \pi\left(2r\left(\frac{dr}{dt}\right)\right)$$

segue...

$$\frac{d}{dx}\left((f(x))^2\right) = 2(f(x))^1(f'(x))$$

4) ONLY Now do I substitute...

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(60\text{ ft})(2\text{ ft/sec})$$

$$= 240\pi \frac{\text{ft}^2}{\text{sec}}$$

KNOW:

$\frac{dr}{dt} = 2$ ft/sec

$r = 60$ ft.

3.6/13) $x^2y + 3xy^3 - x = 3$

Assume that y is a $f^{\underline{n}}$ of x,

$$\frac{d}{dx}\left(x^2y + 3xy^3 - x = 3\right)$$

or...

$$\frac{d}{dx}(x^2y) + 3\frac{d}{dx}(xy^3) - \frac{d}{dx}(x) = \frac{d}{dx}(3)$$

$$(2x)(y) + (x^2)\left(\frac{dy}{dx}\right) + 3\left[(1)(y^3) + (x)\left(3y^2\frac{dy}{dx}\right)\right] - 1 = 0$$

$$x^2\frac{dy}{dx} + 9xy^2\frac{dy}{dx} = 1 - 2xy - 3y^3$$

$$\frac{dy}{dx}(x^2 + 9xy^2) = 1 - 2xy - 3y^3$$

$$\frac{dy}{dx} = \frac{1 - 2xy - 3y^3}{x^2 + 9xy^2}$$